*7th Canadian Conference on Earthquake Engineering / Montreal / 1995*
*7ième Conférence canadienne sur le génie paraséismique / Montréal / 1995*

# Seismic Analysis of Conical Water Tanks

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## ABSTRACT

Conical steel vessels are frequently used as elevated water reservoirs supported on reinforced concrete towers. While design codes of practice guidelines exist for designing against hydrostatic forces, there are no proposed methods for handling earthquake-loads. In this study, the seismic behaviour of liquid-filled conical tanks is investigated using a formulation involving coupling between the finite element and the boundary integral methods. The steel tank is modelled using a newly developed shell element which includes both geometric and material non-linearities, while the hydrodynamic pressure resulting from the fluid-structure interaction is obtained using the boundary integral method. This hydrodynamic pressure leads to a fluid virtual mass which is added to the mass matrix of the structure to perform free vibration and non-linear dynamic analyses of the coupled fluid-structure system. The analysis undertaken shows that the earthquake loading, involving both vertical and horizontal accelerations, has a major effect on the stability of such elevated conical-shaped vessels.

## INTRODUCTION

Conical steel tanks with cylindrical upper section components (Fig.1) are fairly widely used as containment vessels for elevated water tower structures. However, the current standards of practice for water containment structures in North America, based on the AWWA D-100 (1984) specifications, do not provide criteria for the seismic design of such structures.

An extensive number of studies concerning the seismic analysis and design of liquid-filled cylindrical tanks can be found in the literature. The recorded performance of cylindrical tanks during actual earthquakes indicates a common form of buckling of the walls of the tanks near their bases, described as "elephant foot buckling". This localized instability is mainly due to the overturning moment which is exerted by the hydrodynamic pressure resulting from the horizontal component of an earthquake motion. Analytical studies which take into account the fluid-structure interaction and the flexibility of the walls of the tank have been conducted by many investigators. Tanks subjected to horizontal ground motion (Haroun ,1980), and those subjected to vertical acceleration have been considered (Haroun and Tayel, 1985). In the above studies, the boundary integral method was used to obtain the fluid added-mass which simulates the hydrodynamic pressure acting on the walls of the tank as a result of ground acceleration.

To the best of the authors' knowledge, no attempt has been made to study the seismic response of liquid-filled conical tanks. As would be the case for cylindrical vessels, horizontal ground acceleration may be expected to cause significant overturning moment at the base of the vessel. Also, due to the inclination of the walls of the cone, vertical accelerations are expected to induce both axial and hoop stresses in the shell. Indeed, the above studies concerning seismic analysis of cylindrical tanks were limited to linear elastic response, based on the fact that the resulting stresses are usually less than the yield stress of the material. However, for the case of a liquid-filled steel tank, the axial stresses due to hydrostatic pressure when added to those resulting from seismic excitation may indeed lead to yielding. This yielding, together with a localized large deformation near the base could cause premature local buckling in the bottom region of the tank. This could result in an overall instability of the structure. It seems prudent, therefore, to include geometric and material non-linearities when performing seismic analysis of such elevated vessels. The stability analysis of liquid-filled conical tanks subject to seismic loading presented in this study follows a comprehensive investigation of liquid-filled conical tanks under hydrostatic loading presented earlier (El Damatty 1995, El Damatty et al. 1994b).

## FINITE ELEMENT MODEL

A consistent subparametric shell element, free from spurious shear modes normally associated with the isoparametric shell elements was developed by Koziey (1993). This element formulation was then extended by El Damatty (1995) to include large displacements, strain hardening plasticity and non-linear dynamic analysis. Verification of this non-linear model included the non-linear static and dynamic analyses of a number of plate and shell problems (El Damatty 1995 and El Damatty et al. 1994a). In all of the above examples, the finite element model showed excellent performance when comparing results obtained with available analytical and experimental findings in the literature.

The above extended finite element model is now used to model liquid-filled conical steel vessels in order to study their stability under seismic loading. The initial geometric imperfections which may exist in a real shell structure can be represented in the finite element model as initial strain.

## BOUNDARY INTEGRAL FORMULATION OF HYDRODYNAMIC PRESSURE

Two components of hydrodynamic pressure develop inside a liquid-filled tank as a result of seismic excitation. These are the long period component (convective) due to sloshing at the liquid's free surface and the impulsive fluid pressure which varies in-phase with the vibration of the walls. Previous studies of cylindrical tanks indicate that the decoupling between the vibration of the walls and sloshing action is a valid assumption. The same assumption is herein employed for liquid-filled conical tanks. Therefore, only the impulsive hydrodynamic pressure is considered in the analysis. The fluid inside the tank is considered as ideal, while the base of the tank is assumed to be restricted from rocking. In view of the assumptions above, the hydrodynamic pressure resulting from the vibration of a flexible liquid-filled conical vessel filled with water (see Fig.2) is governed by the following equation and boundary conditions:

$$\nabla^2 P_d(r,\theta,z,t)=0 \qquad \text{inside the fluid volume } \Omega \tag{1}$$

$$\frac{\partial P_d(r,\theta,z,t)}{\partial n}=-\rho_F\, \ddot{u}(r,\theta,z,t).n \qquad \text{at the surface } S_1 \tag{2}$$

$$P_d=0 \qquad \text{at the surface } S_3 \tag{3}$$

$$\frac{\partial P_d(t)}{\partial n} = -\rho_F\, \ddot{u}_s(t).n \quad \text{at the surface } S_2 \tag{4}$$

where $P_d$ is the hydrodynamic pressure exerted in the tank in access of the hydrostatic pressure; $\ddot{u}(r,\theta,z,t)$ is the acceleration vector at any point in the tank's walls; n is the unit vector normal to the surface of the tank; $\rho_F$ is the fluid density and $\ddot{u}_s(t)$ is the acceleration vector at the base of the vessel. Surfaces $S_1$, $S_2$ and $S_3$ are as shown in Fig.2.

The above set are solved using the boundary integral method. The general idea is to interpolate the dynamic pressure using shape functions (modes) which satisfy the partial differential equation and also the time independent boundary conditions. The amplitude of each mode is then obtained by satisfying the rest of the time dependent boundary conditions in an integral sense. By applying this approach, the virtual work done by the hydrodynamic pressure $\delta W$ can be expressed in the following manner:

$$\delta W = \{\delta(\Delta U)\}^T [DM] \{\ddot{U}\} \tag{5}$$

where $\{\delta(\Delta U)\}$ and $\{\ddot{U}\}^T$ are the vectors which include the virtual incremental nodal displacements and the total nodal accelerations at time t, respectively. [DM] represents a fluid added-mass matrix which results from the hydrodynamic pressure, while its components include both the pressure shape functions and the interpolation functions of the shell element used to discretize the structure.

The pressure shape functions, due to horizontal acceleration acting on a conical tank which is prevented from rocking, are given by:

$$H_{i1}(r,\theta,z) = I_1(\alpha_i r)\cos(\alpha_i z)\cos(\theta) \tag{6}$$

Meanwhile, the pressure shape functions due to vertical acceleration are given by:

$$H_{io}(r,z) = I_0(\alpha_i r)\cos(\alpha_i z). \tag{7}$$

where $I_1(\alpha_i r)$ and $I_0(\alpha_i r)$ are the modified Bessel's functions of the first order; $\alpha_i=(2i-1)/2\pi h$ where h is the height of the fluid inside the tank. These shape functions are used to obtain the fluid added mass matrices $[DM]_H$ and $[DM]_V$ which results from horizontal and vertical accelerations, respectively. Note that the above added-mass matrices can only be obtained globally and are fully populated. A complete derivation can be found elsewhere (El Damatty, 1995).

## ANALYSIS OF CONICAL TANKS

### Layout and Modelling

Four liquid-filled conical steel tanks (T1 to T4) with constant thickness are considered in the analysis. All tanks have the bottom radius $r_1$=3.0 m, height h=9.0 m and angle $\theta_v$ = 45°, where $\theta_v$ is the angle of inclination of the generator of the tank with the vertical. The thicknesses of the four tanks are equal to 12.0, 12.0, 13.5, and 14.0 mms, respectively. The first tank is assumed to have initial geometric imperfections in the form of a sine wave of wave length equal to the buckling wave length of the perfect structure and an amplitude equal to the tank thickness. The other three tanks are assumed to have a perfect shape. Note that the upper cylindrical segment is omitted in these analyses. This is believed not to affect the results significantly since it is remote from the highly stressed region located at the bottom. The tanks are assumed to rest on four rigid frames. The stiffnesses of these frames are modeled using both horizontal and vertical springs. Based on a preliminary analysis, the springs simulating the frames, which have adequate cross sections to withstand static and seismic loading, were found to be have stiffnesses in the horizontal and vertical directions as: $k_h = 0.708*10^9$ N/m, $k_v = 2.33*10^{10}$ N/m, respectively.

## Static Analyses

Elastic stability analyses (including geometric non-linearities) are first performed for three tanks (T2 to T4) to obtain the buckling wave length due to hydrostatic loading. The buckling displacements are found to be localized at the bottom of the structure. Inelastic stability analysis is then performed for the four tanks filled with water to determine a critical load factor $p_{cr}$ for each tank. This load factor $p_{cr}$ is such that the hydrostatic pressure can be multiplied by any factor up to $p_{cr}$ prior to the tank becoming unsafe. Hence, $p_{cr}$ is a measure of the factor of safety for the tank under static condition when filled with water. The load factors obtained from the finite element analysis are given in Table 1.

## Free Vibration Analyses

The fluid added masses $[DM]_H$ and $[DM]_V$ are calculated following the procedure outlined above. These are simultaneously added to the mass matrix of each tank to obtain the effective mass matrices due to horizontal and vertical excitations. These are incorporated with the linear stiffness matrix of the structure into an eigen value analysis to obtain the natural frequencies and the corresponding mode shapes. The frequencies of the first four modes of vibration due to each type of excitation acting on the tank T2 are given in Table 2.

## Time History Analyses

Non-linear time history analyses of the four conical shaped reservoirs filled with water are performed using the horizontal and vertical components of the 1971 San Fernando earthquake as the input ground motion. The reason for choosing this particular record is that its dominant frequencies contain the fundamental frequencies of vibration of the considered tanks which are given in Table.2. The two components of the acceleration records of the San Fernando earthquake are scaled down such that the maximum velocity of the input record is equal to the zonal velocity of Quebec city as specified in the NBCC (1990). This leads to a maximum horizontal and vertical acceleration equal to 0.28 g and 0.167 g, respectively, where g is the acceleration due to gravity. Only the strongest six seconds of record are used in the analyses because of the very long computer time associated with this type of time history analysis problem. The scaled horizontal and vertical accelerations of the earthquake are shown in Figs.3 and 4. Due to symmetry about the direction of the horizontal excitation, only one half of the tanks is modeled in the analysis.

The fluid added masses $[DM]_H$ and $[DM]_V$ as well as the mass matrix of the structure are added together to obtain the effective mass matrix $[M^*]$. The non-linear stiffness matrix of the structure includes the effect of both the geometric and the material non-linearities. A 2% viscous damping value for the liquid-shell system is assumed in the analysis. The time history analysis is achieved using the Newmark method for time integration and the Newton-Raphson method for iteration within time increments equal to 0.02 sec.

The results of the dynamic analyses are presented at different locations on the tank circumference. $\theta=0°$ and $\theta=180°$ are located on the axis of horizontal excitation, while $\theta=90°$ is located in a direction perpendicular to the axis of horizontal excitation. The transverse meridional displacements along the generator ($\theta^\circ=0$) of tank T1, which failed during the time history of the earthquake, are displayed in Fig.5. In this figure, the dotted plot represents the displacement shape resulting from the hydrostatic pressure, while the solid line shows the displacements just prior to dynamic instability. From the plots it can be observed that the dynamic buckling which is localized at the bottom of the tank has the same pattern as that of the static displacements. Also to be noted, are the large horizontal and vertical movements in the upper region of the tank due to the seismic motion. In Fig.6, the same displacement plots are plotted along the generator ($\theta=180^\circ$) of the tank. This latter figure shows no evidence of buckling along that generator. This means that the buckling is localized near the base and is confined to the region subjected to high compressive axial stresses resulting from the overturning moment. The plots of the displacement at different locations for the tank T4, which survived the simulated earthquake without any inelastic behaviour, shows no out of roundness effect at any section of the tank. The relative displacements along the x-axis (axis of horizontal excitation) at the top section of the tank is shown in Fig.7. Meanwhile, the relative vertical displacements at $\theta=0^\circ$ and $\theta=90^\circ$ are plotted in Figs. 8 and 9. It is important to point out that the response at $\theta=90^\circ$ is only due to vertical acceleration, while the response at $\theta=0^\circ$ results from both horizontal and vertical accelerations. The meridional stresses in the bottom section of the tank are plotted for $\theta=0^\circ$ and $\theta=90^\circ$ in Figs.10 and 11, respectively. Similar to vertical displacements, the results for stresses at $\theta=90^\circ$ are only due to vertical accelerations, while those at $\theta=0^\circ$ are due to both the vertical and horizontal components. Note that in all of the above plots, the response at t=0 corresponds to the effect of the hydrostatic pressure before applying the seismic loading. From the plots of the stresses, it can be observed that the maximum stresses induced at the bottom by the vertical acceleration are almost 32% of the maximum stresses induced in the same section by the horizontal acceleration. It can also be observed from the plots that the stresses in the critical region due to the seismic motion are larger than those resulting from the hydrostatic pressure. The results from the time history analysis are summarized in Table 1. The last column denotes the most critical state experienced by the structure during the six seconds of record. The term "safe" denotes that the tank has survived this earthquake motion, while the tanks which have suffered from dynamic instability during the six seconds are described by the term "failed". In the same column, tanks which have a complete elastic response during the record are described by "elastic", while the term "plastification" denotes the tanks which have an inelastic response during the seismic motion. To obtain a fully elastic response and assure the safety of the stucture from dynamic instability, the results of the dynamic analyses show that under a seismic excitation having frequency content of the fundamental modes and a maximum acceleration of 0.28 g, a load factor under static conditions of 2.8 has to be provided. Meanwhile, a load factor of 2.65 leads to a safe inelastic response of the tall tank under the same excitation.

## CONCLUSIONS

From the stability analyses of liquid-filled elevated conical tanks subjected to seismic loading undertaken in this study, the following conclusions can be drawn:

1) Tanks which apparently have a high static load factor may exhibit inelastic behaviour during an earthquake record, followed by inelastic localized buckling near the base of the tank. Therefore, a proper modelling procedure along with time dependent analysis must be followed in order to design such tanks safely. The model developed here is meant to satisfy such a need.
2) The vertical component of ground acceleration does contribute significantly to the dynamic instability of liquid-filled conical vessels and can not be ignored in a seismic analysis of such a structure.

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Table 1 Results of the Time History Analyses for Conical Tanks.

| Tank | $p_{cr}$ | Results Description | Tank | $p_{cr}$ | Results Description |
|---|---|---|---|---|---|
| T1 | 1.4 | Failed (Plastic) | T3 | 2.65 | Safe (Plastic) |
| T2 | 2.25 | Failed (Plastic) | T4 | 2.80 | Safe (Elastic) |

Table 2 Natural Frequencies (cps) of Tank T2.

| Type of Excitation | Mode 1 | Mode 2 | Mode 3 | Mode 4 |
|---|---|---|---|---|
| Horizontal | 2.51 | 3.54 | 6.67 | 12.04 |
| Vertical | 7.44 | 14.95 | 19.06 | 24.46 |

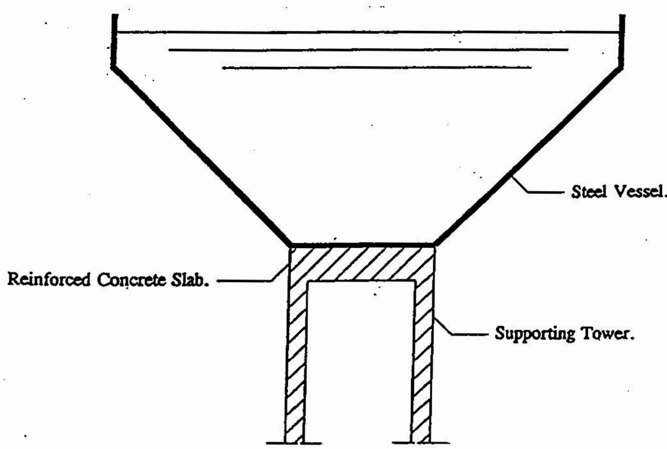


Fig.1 Cross Sectional Elevation of Elevated Conical Tanks.

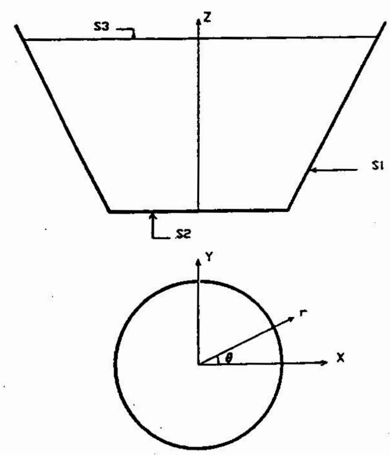


Fig.2 Coordinate System.

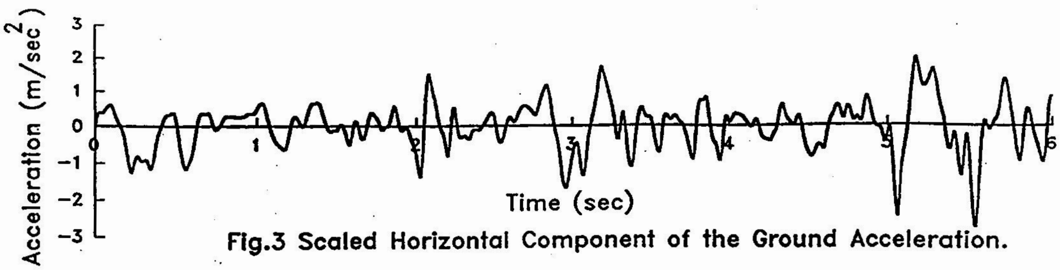

**Fig.3 Scaled Horizontal Component of the Ground Acceleration.**

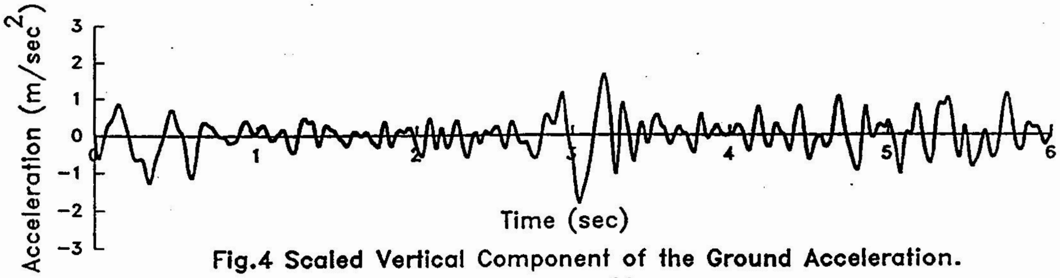

**Fig.4 Scaled Vertical Component of the Ground Acceleration.**

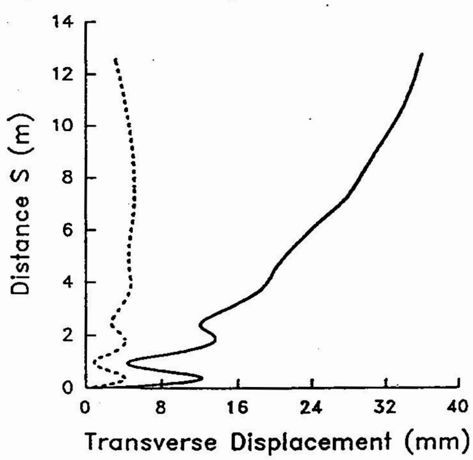

Fig.5 Displacement Along ($\theta$=0) For T1.

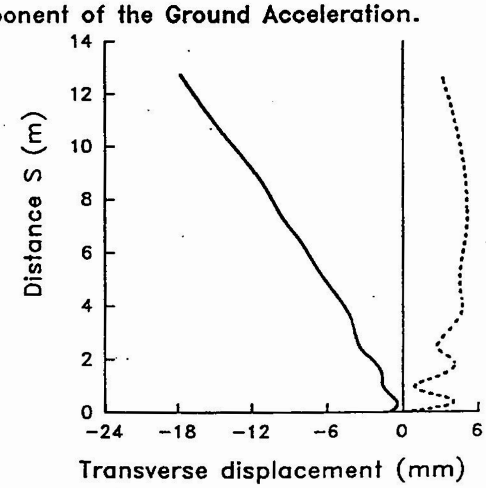

Fig.6 Displacement Along ($\theta$=180) for T1.

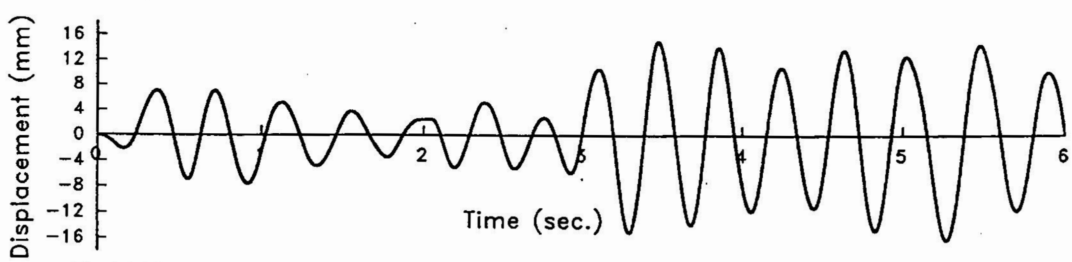

Fig.7 Time History of the Relative Displacement Along the X–Axis At Top of T4.

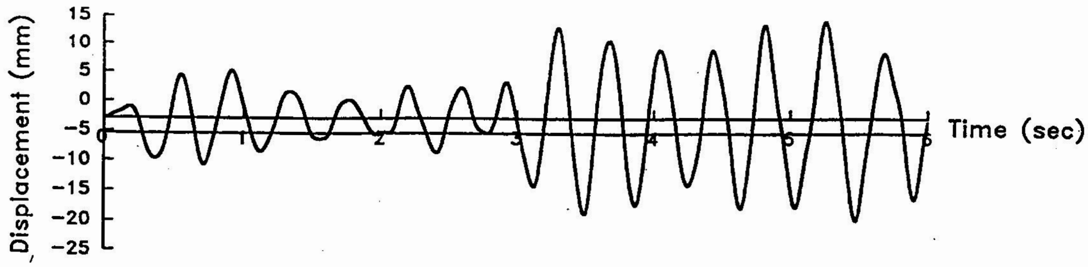


Fig.8 Time History of the Relative Vertical Displacement At Top of T4 ($\theta$=0).

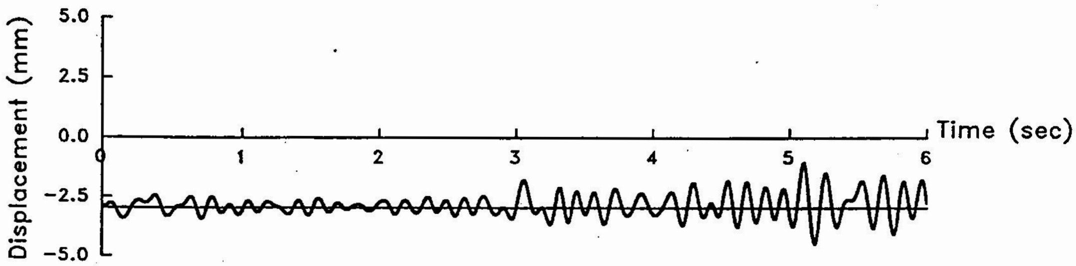


Fig.9 Time History of the Relative Vertical Displacement At Top of T4 ($\theta$=90).

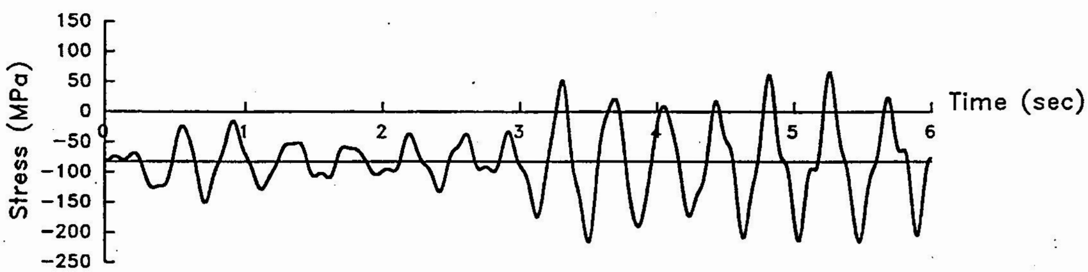


Fig.10 Time History of the Meridional Strsses at Bottom of T4 ($\theta$=0).

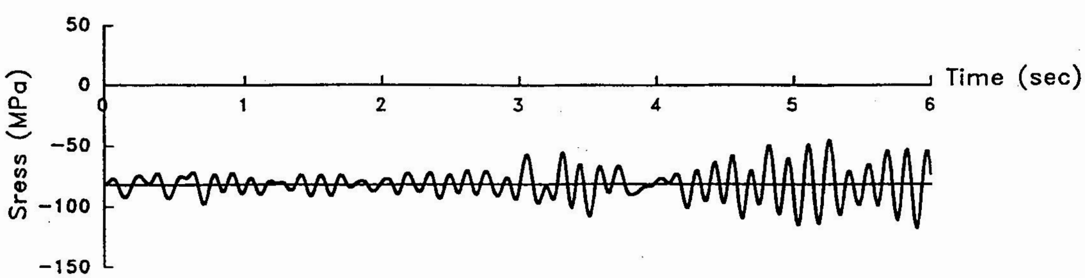


Fig.11 Time History of the Meridional Stresses at Bottom of T4 ($\theta$=90).